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Understanding complexity across scales and levels

A Literature Review prepared for DSTG

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Executive Summary

Decision-making in the modern world is becoming more difficult across all types of organisations, from governments and agencies to small businesses and charities. Issues and events from previously distinct spheres are increasingly overlapping with unexpected consequences evolving rapidly. For example, over the past few years geopolitics has become increasingly important for grocery supply chains. The complexity of issues and systems that need to be understood has risen rapidly, alongside the speed at which influence, change and feedback propagate through these systems. We no longer have the luxury of time that often existed in the past.

These challenges are well-known, including providing much of the context in the 2020 Defence Strategic Update, yet addressing them remains difficult. This literature review seeks to tackle these issues from a different perspective, by considering them as a challenge to our analytic and research practices, not just our decisions or activities. Our habits of thinking and analytic practices, especially in large organisations, were honed when we had more time and less complexity to consider. One significant reason that decisions aren't keeping pace with reality is that ability to understand what is going on is also lagging.

Complexity is a factor that researchers and analysts have always had to grapple with, and developments in complexity science over recent decades have provided an explicit vocabulary and set of concepts to help deal with it. However, despite the increasing complexity we are facing, few organisations or analysts effectively incorporate complex systems thinking into their work.

This literature review is one stage in a broader project, funded by DSTG, to help bridge this gap. It has scanned a wide range of academic literature to identify methods that researchers have used to analyse and make sense of complexity and complex systems. Notably, a core assumption in the review is that, while complexity science gives us a useful set of concepts, research investigations of complex systems are much wider than just complexity science. For this reason, the review has cast a broad net.

Given the purpose and funding source of the broader research project, this literature review has narrowed its scope to a particular context: large organisations that split responsibilities across different functions, with analysis significantly separate from decision-making. This shaped the research in a few important ways.

Firstly, most of the common frameworks developed within complexity science are a difficult fit for this organisational context. These are typically forms of decision frameworks to help integrate complexity considerations into decision-making. Good examples include frameworks like soft systems methodology, critical systems heuristics and Cynefin. However, decision frameworks aren't that useful for analysts who don't make the decisions, while the complexity frameworks are beyond the current bandwidth of busy decision-makers. The focus of this review is on assisting analysts, and so the methods covered are focused on understanding complex systems, rather than acting within them.

Secondly, as a consequence of this, an alternative framework needed to be developed to help structure and sequence the range of methods uncovered. The motivation for and structure of this framework is covered in the first two sections of this review.

Thirdly, it should be noted that the incentives around academic publication complicated the task at hand. Research publication focuses on the final results, whereas our interest was largely on the methods by which people got to the results. These could, in many cases, be identified or inferred. However, it isn't as simple as understanding the research method, as often the most pertinent question is how researchers decided on a particular method. For these reasons, the results in the review cannot be taken as definitive and further research would be beneficial.

The literature review is divided into four sections and includes a substantial reference Appendices.

The first section sets out core definitions around complexity and complex systems. This focuses on two key features that produce complexity and mean that complex systems cannot be analysed in traditional reductive ways. One is the existence of feedback loops or mutual dependencies between different parts of the system. The other is that causation and influence are not restricted to entities or organisations of comparable scale or level of complexity/abstraction.

The second section builds on this to identify key principles for thinking about complexity around a practical process that traces through analysis to decisions:

1. **Scope:** *Identify the purpose of analysis and the boundaries around the relevant complex system.*
2. **Explore:** *Seek to understand key system features and identify expected system behaviours*.
3. **Build:** *Capture insights in a coherent structure for communication, prediction, testing and further insight.*
4. **Apply:** *Use the description for testing, evidence or insight in decision-making.*

While this approach is more broadly applicable, it is primarily designed for decision-making systems where the analysis and research are conducted separately from separate decision-making processes. In other words, the *Apply* stage is the responsibility of a different set of people to the previous stages. For that reason, it has not been covered in any detail here.

The third and fourth sections cover a wide range of methods identified through the literature review, divided into whether they are most useful within the *Explore* or *Build* phases in the process. More focus was applied to methods relevant for the *Explore* phase as this is generally less intuitive and is not as well understood. The methods in the two sections are prioritised by expected applicability in a general analytic setting. The Reference Appendices include more details on the methods, including strengths or weaknesses and references to research papers.

The findings of this literature review will be refined through experimentation and testing before a final report is produced. This will narrow down the prioritised methods to a shorter list that can be more easily adopted by a wide range of researchers or analysts.

An Introduction to Complex Systems

Analysis across a range of fields, whether for policy, intelligence, operations or foresighting, necessarily grapples with large, interconnected systems of actors and entities. These systems are commonly identified as *complex systems* that can be described in various ways by complexity science (Dent, 1999). However, the various frameworks and approaches within this field are rarely applied and even more rarely understood by analysts across government, business or even in other areas of research. Complexity science is often conceptually dense and seems counter-intuitive to those trained in a range of ordinary approaches to analysis.

Common to research disciplines, organisational structures and many methods of analysis is the assumption that the best way to solve big problems is to break them down into smaller parts and solve the smaller problems. While this reductionist approach has been incredibly successful in many situations, it depends on influence or causation flowing in one direction – from the smaller parts into the larger ones. This approach makes sense in chemistry (tracing chemical properties back to molecules and then atoms) or the analysis of financial flows or accounts (the headline figures are entirely composed of lots of small transactions). However, there are many situations where it doesn't.

Let's look at a topical question in 2022: for any given country, who determines the country's attitude towards the war in Ukraine? Is the government, the population, the media, the bureaucracy, the intelligentsia or someone else? The government will articulate an official position, but this is normally formulated with assistance from the bureaucracy and perhaps others. This position, however, has to be reasonably consistent with public and media attitudes otherwise there will be protests and other consequences. Yet public attitudes are rarely static and are often influenced by government announcements and the views of others.

Influence in the national systems that determine foreign policy positions flows in multiple directions across lots of groups and actors. An analysis that breaks the system down to look at the different groups individually will miss the real, and multi-faceted, dynamics of the way influence flows across groups. As influence flows in many directions, this is a system that can't be fully understood by breaking it down into parts and is, therefore, a good example of a complex system.

How are complex systems different?

This idea, that a complex system can't be fully understood by breaking it down into parts, illustrates the problem but doesn't explain why it arises. To do that we need to understand what makes systems complex (in a somewhat technical sense) rather than just thinking of them as messy, intricate or complicated. Importantly, this isn't the size of the system or the number of entities involved, but rather the way that different parts of or entities within the system are related. There are immense systems that are simply complicated, not complex (Kamensky, 2011). We have identified two key features that usefully explain what makes systems *complex*.

The literature on complexity often emphasises 'feedback loops' as a key feature (Kastens et al., 2009). A feedback loop occurs between two (or more) entities in a system when a change in one causes a change in the second but this in turn causes a further change in the original entity. In the example above, a change in government policy might lead to a change in public attitudes that in turn causes government policy to change again.

However, the concept of a 'feedback loop' doesn't truly capture complexity. It suggests we can identify which thing changed first and started the loop. A more relevant, but similar, way of thinking about complex systems is that it includes multiple entities which have mutual dependencies.[1] That is, these entities depend in some way on the other entities and there is generally no clear way of saying what comes first.

An ecological example can illustrate this neatly. In an ecosystem, the amount of vegetation and size of the plant-eating animal population are mutually dependent. More animals reduce the vegetation, but less vegetation reduces the number of animals that can live there. We can't say which of these is primary as both depend on each other. To put it differently, we can't understand the system by breaking it into separate studies of vegetation and of animals.

Similarly, in our previous example, there are commonly mutual dependencies between media reporting and public attitudes, between public attitudes and government policies, and various other groups. Asking which comes first misses the point. These mutual dependencies mean that analysis of a change in the system typically won't be able to point to a single cause or factor. It is often multiple things working together via mutual dependencies and feedback loops.

A second common characteristic of complex systems is that influence or causation happens across scales or levels of organisation. Systems are very often made up of many different entities that differ in size or physical scale (e.g., molecules versus biological organisms; individual people versus nations), internal organisational complexity (e.g., a table and a human being are similar sizes but have different types of internal organisation), or conceptual abstraction (e.g., national economic dynamics are different to individual finances).

In chemistry, for example, atoms act on other atoms, molecules act on other molecules and so on. An atom only acts on a higher level entity, say a biological organism, by means of acting on atoms in that structure. Complex systems aren't neatly hierarchical like this, and causation or influence can happen directly across levels. For example, in a national system, national policy or legislation directly affects individual people and periodically, individuals directly change national policy (including from outside the normal decision makers).

When a system includes mutual dependencies and causation across scales and levels, seeking to analyse it by reducing it into component parts will miss critical features of the system. Thus, complex systems require us to think differently about how we analyse and understand them.

It is worth noting that human, social systems almost invariably exhibit these two characteristics. For a start, humans are influenced by a wide range of factors across wildly divergent scales - from molecules (like drugs) to landscapes, from other people to ideas, laws and institutions. This diversity of influence and causation across scales immediately creates complexity as it isn't possible to break down social systems into discrete parts.

Human relationships are also regularly characterised by mutual dependencies. A simple, and very common, example is what happens when two people negotiate – whether it be about a price, a meeting time or something more complicated like a game or agreed set of work. If we ask, 'who set the price or chose the time', the answer is very often that they both did. Each person's preference is influenced by what they expect from the other person, but then the negotiation proceeds dynamically based on each reacting to the other. There are clear feedback loops without any starting points, hence mutual dependencies.

Complex systems behave differently

Given that complex systems have mutual dependencies between parts, and that causation and influence happens in lots of directions and across scales, they tend to exhibit different properties and behaviour than other systems. Some commonly identified features, expressed by the concepts of complexity science, are emergence, nonlinearity, self-organisation and universality (Ladyman et. al., 2013; Siegenfeld and Bar-Yam, 2020). Notably, these features do

[1] Terminology here varies and interdependencies is sometimes preferred. For example, see MacKay 2008.

not define complex systems as they can arise in other situations. However, they are common in complex systems and are therefore useful heuristics to help understand and predict complex system behaviour.

Emergence is the idea that “the whole is greater than the sum of its parts”. That is, it is the idea that a complex system can exhibit properties or behaviours that cannot be observed in its constituent elements. Examples of emergent behaviour include the flight patterns of swallows, mounds of termites or crowd behaviour in humans.

Self-organisation refers to the way in which certain forms of order sometimes arise from local interactions between parts of an initially disordered system, without any overall guiding direction. One example of self-organization includes the approximate radial symmetry of snowflakes, which arises from local attractive and repulsive forces between water molecules and the surrounding environment.

Another commonly cited feature of complex systems is nonlinearity. Nonlinearity typically means cases where the output of a system is not obviously a direct function of the inputs (Willy et. al., 2003). Intuitively, it is often useful to understand this in terms of stability. In the right circumstances, complex systems can be very fragile. A small change or small input can lead to a dramatic reorganisation or collapse of the system, way out of proportion to the size of the input. The assassination of a single prince on the periphery of Western Europe in 1914 is a memorable example.

Nonlinearity can also work in the opposite direction, as complex systems are often highly stable and resistant to change. Complex systems often absorb a range of inputs and return to the status quo ex-ante, even though the inputs may suggest a sizeable shock to the system.

More broadly, the properties of complex systems often overlap with a wide range of classes of mathematical nonlinearity, such as chaos theory and certain sets of differential equations. These systems are highly sensitive to initial conditions and are therefore also sensitive to historical variation. As a result, even if a complex system returns to a state similar to one it held previously, the same input may provoke a radically different reaction.

The main point to note is that complex systems are very difficult to model numerically. Many of our numerical intuitions, which tend to be based on standard linear statistics (e.g., bell curves and significance tests), are unreliable as guides to thinking about complex systems.

Finally, complex systems often exhibit universality. Originating in statistical mechanics, universality refers to patterns of behaviour that occur across multiple spatial and temporal scales and which are substrate neutral. In other words, complexity can give rise to very similar patterns at very different levels of organisation due to similarities in the structures of systems and patterns of dependencies. Universality means that we can potentially look for insights into the behaviours of nations in the behaviour of individual people (or even animals). But it also means that we can’t make sense of these systems via reductionist approaches.

Different ways of thinking

There is something of a paradox in how we structure and conceptualise analytic processes. Humans have always been embedded in and sought to understand complex systems, particularly social and political systems. Thus, while complexity science is a relatively recent field, humans have always been trying to understand and make sense of complex systems. However, we tend to think and structure work in reductionist ways that make it difficult to grasp the inherent complexity we are trying to understand.

Diagnosing the origins of this paradox is beyond this work and we are more interested in remedies. Experience has shown, however, that one obvious direct remedy – teaching complexity science – has not made a significant impact on day-to-day analytic approaches, whether that is in academic research or policy and intelligence analysis. What we have synthesised here is more indirect but doesn't depend on the conceptual overheads required to understand complexity science.

There are three different parts to this approach. While each is valuable by itself, they will likely work best when applied together. Firstly, we have three key principles to guide exploration of complex systems and summarise important differences to traditional research approaches. Secondly, we have proposed a high-level workflow that, while widely applicable, is particularly relevant for complex systems. The third part is a set of methods or techniques that help make sense of complexity. At this stage, this is a long list derived from various literature that will be tested and prioritised for the final report.

Principles

Firstly, as a result of the types of dependencies involved and the likelihood of properties like emergence and non-linearity, in general, **complex systems are not definitively solvable**. That is, there is no method that can be applied to a complex system that will completely describe the system and answer all likely questions we will have about it. Some less intricate complex systems may be fully describable for relevant purposes using specific techniques, but this won't hold in general.

In almost all situations, especially where there are time and resource constraints, this means that *we can only expect to find partial descriptions of a real complex system*. Importantly, if we are seeking to reduce a complex system to a single underlying structure, we will fail to fully describe the system as we will be leaving important features out.

Secondly, as complex systems have causation and influence flowing in multiple different directions across different scales and structures, **understanding complex systems involves bring different sources of information and ways of seeing a system together**. We need to synthesise multiple sources into a coherent view, rather than breaking it down via analysis. There is no single way of analysing them that will give us the answers we want.

There are a couple of immediate and important consequences to this. One is that consciously incorporating a diversity of inputs – sources, disciplines, perspectives, thinking styles – is important to understanding many complex systems. This is often difficult in practice, but highly productive when done well. A second is that consciously adopting multiple different analytic approaches or frameworks helps us understand complex systems better. Using different approaches often reveals a different aspect of the system that we would miss otherwise.

Thirdly, as complex systems operate as integrated wholes, rather than collections of discrete parts, to understand them **we need to focus on building rather than dissecting**. That is, we can't cut up a complex system into its parts to understand how it works as that immediately removes the key features of complexity. Instead, we need to try to build systems that exhibit similar properties to see how the complex system operates in reality. These built systems can be everything from mental models or stories to computer simulations or physical models. An important caveat is that this process should ideally be iterative. Complexity is hard to get right (especially due to emergence and nonlinearity) so we need to continually test what we have built against the real system to check how accurate it is.

Workflow or process

Taken together, these principles suggest that certain ways of working will be more effective when working on or with complex systems. These will not be unique to complex systems – we don't need to do everything differently – but there are particular reasons why these matter for complexity. We have synthesised these into a four-phase workflow that is designed for practical decision-making, especially in organisations where different units are responsible for different parts of the process.

The four phases are four types of work that need to be done, broadly in the order specified, but in practice, it will never be a simple linear process. At many points, you may want to loop back to earlier stages and revisit or tweak the work. These phases are also highly scalable – it is possible to touch all of them in a few hours or build them into a multi-year research project.

These four phases are (with more detailed descriptions following):

1. Scope: *Identify the purpose of analysis and the boundaries around the relevant complex system.*
2. Explore: *Seek to understand key system features and identify expected system behaviours.*
3. Build: *Capture insights in a coherent structure for communication, prediction, testing and further insight.*
4. Apply: *Use the description for testing, evidence or insight in decision-making.*

Scope

As noted, complex systems are rarely definitively solvable. Moreover, in the real world, there are rarely discrete demarcations between different complex systems. For example, ecology blurs into weather, geology and increasingly sociology. This means that seeking to understand a particular complex system writ large will almost invariably lead to an indefinitely expanding task. We need therefore to narrow our analysis and put clear boundaries around it, otherwise, it will never be achievable.

To effectively draw boundaries around the analysis and the system we are interested in, it is useful to know the reason why the analysis is being performed and what purposes it will be used for. To look again at the example of a country's attitude towards the war in Ukraine, the boundaries of the system we are interested in will vary depending on if we are interested in the effectiveness of international sanctions regimes or whether we are looking at potential unrest in that country. As these systems are complex and interconnected, we cannot look at everything, so we need to start with as clear a scope as we can. Where needed, it can be updated as you go.

Explore

Due to the many challenges involved in understanding complex systems and especially since they aren't easily solvable, it is important to take time to try to understand how the system functions in practice and make sense of its properties. This might sound overly simple, but it is a common practice for analysts to jump straight to writing products or for researchers to begin by building models. This can be sufficient when we have an existing approach that we know will work for the problem or system involved, but in every other situation it will likely simplify out important features of complexity and lead to a highly incomplete analysis.

This phase is where synthesis and diversity are critical to getting a more complete understanding of the system. Even in the rare case that we have an analytic approach that is likely to solve the system for our purposes, it is useful to consider multiple perspectives on it and take multiple analytic approaches to ensure that our preferred approach isn't missing something important. The more diverse the perspectives, sources of information and analytic methods we can bring to the problem and synthesise, the more robust our exploration.

However, in practice, there is almost invariably a significant trade-off between diversity and timely delivery. It is also often very hard to do due to lots of human factors. A practical rule is generally to introduce as much diversity as possible without breaking or delaying the work. Some diversity is always better than none.

Build

The output of the exploration of a complex system is, at least initially, likely to be a series of observations of system behaviour under certain conditions. While these are often valuable by themselves, they are both hard to apply within decision-making and fall short of a coherent understanding of the situation as an interconnected system. It is therefore important to build a description, or descriptions, of the system that treat it as some kind of whole.

There are a wide range of ways this can be done, and the best method will often be dictated by the purpose of the work, how it will be applied and who you are working with. It can involve computer modelling, games and simulations, writing scenarios or even crafting relevant metaphors. What is important is that these descriptions are built and then tested iteratively, both for accuracy and usefulness.

Apply

The whole process, including building the understanding of the system, will typically be designed to feed into some decision or action. So this is the point at which complex system analysis joins into ordinary decision-making processes, which are highly varied. This review has deliberately not investigated this part of the process due to the types of organisations that it is focused on. However, a key principle is that all the prior work should be designed to ensure that it has the right effect and is useful at this stage. High quality analysis is often ignored in the actual decision making as the outputs have not been produced in a way that works with application and decision making within the relevant organisation.

Methods for exploring complexity

Through the literature review, ten different methods were identified that researchers have used to seek to understand or explore complex systems across a wide range of literature. A very short summary of each of the methods is listed below, divided into two sections.

The first includes a short list of six methods that we suspect can have immediate application within organisational analytic functions. These will be tested and prioritised in a second phase of the project for usefulness and applicability. The remaining list is included as a reference point for those who may be interested. More details on all of the methods, including references to literature that use the method, are provided in Appendix A.

Readily applicable methods

Comprehensive mapping at a scale (Horizontal slice): Seek to identify all the relevant factors at the scale you are interested in (e.g., national factors for a national analysis) and comprehensively map their relationships.

Key factors / nodes: Analyse the system based on interactions and system dynamics of a select number of identifiable key factors, drivers or nodes in the system – as a way of simplifying a comprehensive map and focusing on key dynamics.

"In the life of….": focus on an individual entity within the system (e.g., person, animal, place, molecule, business, etc) and trace their real or likely journey over time, identifying the influences and forces that affect them, their decisions and their interests.

Break the system: put the complex system under stress (real or simulated) and try to break it. Seeing what happens will help identify important system dynamics across different scales.

Random selection: pick out a small, tractable number of factors or entities that are widely dispersed across the whole complex system and map their relationships (possibly through intermediaries) to build a different picture of the system and cross-scale dynamics.

Empirical conceptual analysis: Complexity is often present in how different people understand a system, not just the system itself. Conceptual analysis seeks to make explicit the features of how different people understand terms and definitions and therefore how we apply them. Gaining clarity on terms, via empirical testing not just thinking about it, often yields important insights about systems that are obscured by fuzzy definitions or overlapping concepts.

More difficult methods

Comprehensive mapping: Seek to identify all the relevant factors across all relevant scales and comprehensively map their relationships.

Emergence: focus the analysis on one level down, or one smaller scale, to identify the emergent system's properties at that level which will drive the important dynamics at the scale of interest. (e.g., focus on local and internal factors that will produce emergent dynamics at the national scale)

Property dynamics: Identify key properties of system elements that are widely shared and track these to map, characterise and understand the dynamics of the system. Examples of relevant properties might be information, entropy, workload, or debt.

Complexity watch: examine a system and consciously look for typical complex system behaviours, such as emergence, non-linear dynamics, tipping points and stability/memory. Where these are identified, you can work backwards to understand the system leading to them or focus analysis at the relevant level of the system.

Methods for building or modelling complexity

There are a wide range of ways we can build an understanding of a complex system: some are highly intuitive for humans as we have been grappling with this for millennia, while others are highly technical. Given the incentives at play, the research literature has largely focused on technical approaches, mostly through a wide range of formal modelling approaches to complex systems.

Given the number of methods and volume of literature, the list of methods below reflects an initial literature scan to pull out of a representative collection of methods. Again, it has been divided into those that are likely to be more immediately applicable by a general analysis and those that are more complex and/or require specialist training or skills. For this reason, the first category is generally more discursive approaches, and the latter typically involve formal modelling. References and examples for these are included in Appendix B.

Discursive methods

General Principles: A simple way to capture and communicate the behaviour of a complex system can be to articulate a number of general (but likely not universal) principles that explain typical system behaviour. Most academic fields that grapple implicitly with complex systems articulate their findings in this way.

Metaphors: Very disparate complex systems often have similar properties or behave in similar ways due to similar system structures. This means using metaphors that explain unfamiliar systems in terms of familiar ones can intuitively capture insights and system properties.

Scenarios: Capture insights about how the system will work in a number of discrete stories or scenarios. Stories intuitively rely on tensions and complexity. These can be focused on different potential system states, or different ways that the system could evolve.

Games or Simulations: Codify key insights about a system as rules of a game and get people with relevant expertise to play out the game as a way of modelling the system, communicating features and building greater understanding.

Technical modelling approaches

Agent-based models: Focus on the individual entities or agents involved in a system and build a model based on their states, the environment they exist in, and the laws that govern agent interactions.

Network Models: This approach focuses exclusively on the properties of relationships between entities in the system, typically constructed as a network. Key network parameters can capture important structural features of a complex system.

Power Law Distributions: Empirically, complex system behaviour often reflects various mathematical power laws, which in turn provide a useful way of mathematically modelling different systems with similar structures.

Differential and Recurrence Equations: Dynamics models of systems that capture the evolution of properties over time are often built on differential or recurrence equations. Differential equations deal with continuous, smooth processes, whereas recurrence equations deal with discrete, stepwise processes.

Subsystem modelling: An effective way of simplifying the modelling process is to focus on the interactions between subsystems with their own (emergent) properties, rather than one connected system. This might build on models of the subsystems, or just emergent properties.

Reference Appendix A: Detailed overview of techniques

This Appendix contains more details on the techniques covered in previous sections, including strengths and weaknesses in application. It is expected to be used as a reference section as needed.

Comprehensive mapping at a scale (Horizontal slice)

Overview

Where system dynamics are understood (or believed) to be relatively stable at a particular scale, restricting system exploration to that scale makes the task more tractable. A common approach is to identify as many relevant factors and entities as possible at the scale you are interested in (e.g. national factors for a national analysis) and comprehensively map their relationships. This is simpler than attempting a full systems map. If we adopt the usual visual metaphor where different scales or levels interact vertically, then this approach focuses on taking a horizontal slice of the system and examining that in detail.

In one good example, Hidalgo and Hausmann (2009) take this approach in looking at how the complexity of a country's economy can be characterised by its trade network. This analysis maps the network of trade solely at the international level. Different countries are taken simply as simple units. The research found that the complexity of a country's economy can be described by the relationships between the ubiquity and diversification of goods, as well as a nations nontradable "capabilities". The findings agree with historical income and growth trends.

Insights

Given that the difficulties of mapping a system increase dramatically with greater granularity, this approach can provide a sensible constraint that still produces useful insights without requiring as much investment. However, this constraint is often not sufficient to reduce the analysis to something easily achievable, especially in the absence of a long research program.

This approach is comfortable for traditional methods of research, which have tended to focus at particular scales. For example, psychology focuses on humans as individuals whereas strategic studies focuses on countries. As such, however, it will likely not reveal many of the more unexpected or dynamic features of complexity that emerge across scales and levels.

It will deliver the best results where the relevant system you are looking at is well-defined at the particular level you are interested in. It is also important that the entities at that level have clear and direct relationships with other entities at the same level

Example Literature

Hidalgo, César A., and Ricardo Hausmann. 2009. 'The Building Blocks of Economic Complexity'. *Proceedings of the National Academy of Sciences* 106(26):10570–75. https://doi.org/10.1073/pnas.0900943106

Key factors / nodes

Overview

While researchers will often want to map a system comprehensively, there are generally significant data, discipline and resource limitations. A natural way to manage this is to limit the exploration of a system to key factors, entities or nodes that either are most important or for which you have most information. This can be a sensible way of making the analysis more tractable without eliminating the complexity.

In recent research, this approach has been adopted to make sense of large data sets. One good example is Alfano et. al. (2021), who used the 2020 BLM movement on Twitter as a case study to measure the functionality of hashtags and emoji in online activism. They argue that authors (*system nodes*) who use hashtags and emoji sacrifice their immediate engagement (*edges*) to signal ingroup loyalty. A similar approach is adopted by Quintana et. al. (Preprint).

The approach has also been core to various mathematical approaches, built on graph theory and topology. For example, Ravasz and Barabási (2003) exploit the *clustering* abilities of *scale-free* systems to produce a hierarchical network model. This is particularly useful for the study of quantifiable and discrete systems, as well as systems with topological issues.

The approach is also used in a less formal or more explicitly methodological way. Funke (2021), for example, provides a conceptual framework composed of five key features. This framework is useful for understanding human behaviour in a social context.

Insights

The major advantage of this approach is allowing a wide range of techniques to be applied to a simplified, and therefore more tractable, version of a complex system. Where good data exists, this often allows us to apply many powerful statistical, visualisation and mathematical methods. However, good data for what we want rarely exists and this can end up with us looking at the data we have for answers that it can't provide.

For other types of systems, if we have an intuitive grasp on the key factors, or an easy way to generate this, then it provides a nice framework to develop a useful, albeit simplified, understanding of the system. The simplified view can be potentially somewhat mitigated by exploring the system through different configurations of key factors and nodes, as each will likely reveal different information about the system.

Example Literature

Alfano, Mark, Ritsaart Reimann, Ignacio Quintana, Marc Cheong, and Colin Klein. 2021. 'The Affiliative Use of Emoji and Hashtags in the Black Lives Matter Movement: A Twitter Case Study'. https://doi.org/10.21203/rs.3.rs-741674/v1.

Funke, Joachim. 2012. 'Complex Problem Solving'. Pp. 682–85 in *Encyclopedia of the Sciences of Learning*, edited by N. M. Seel. Boston, MA: Springer US. https://doi.org/10.1093/cz/zoz016.

Quintana, Ignacio Ojea, Marc Cheong, Mark Alfano, Ritsaart Reimann, and Colin Klein. 2022. 'Automated Clustering of COVID-19 Anti-Vaccine Discourse on Twitter'. https://doi.org/10.48550/ARXIV.2203.01549.

Ravasz, Erzsébet, and Albert-László Barabási. 2003. 'Hierarchical Organization in Complex Networks'. *Physical Review E* 67(2):026112. https://doi.org/10.1103/PhysRevE.67.026112.

In the life of….

Overview

Enbedded within long-standing research approaches such as ethnographic research and grounded theory is a distinct approach to understanding complex systems. Rather than trying to understand the system as a whole, narrow the focus to individual entities within the system and follow their experiences navigating the complexity. This approach of focussing on an individual entity within the system (e.g. person, animal, place, molecule, business, etc) and tracing their real or likely journey over time, allows you to identify broader system influences and forces that affect them, their decisions and their interests. This, especially where the approach is applied to multiple dispersed entities within a system, provides an intuitive way to explore a system that builds a useful understanding of the complexity.

A classic example of this approach is found in Geertz (1972), who adopted a version of this methodology to understand the social dynamics around cockfighting in traditional Balinese culture. Geertz used an interpretive approach which aims to situate observations in their broader cultural and semiotic context, as opposed to the more traditional ethnographic approach of systematically collecting putatively objective facts. This generated a qualitatively different and powerfully insightful picture of the social dynamics at play.

A different approach is found in Apgar, Argumedo and Allen (2009) who examined a case study of the Kuna and Quechua indigenous peoples to understand their lived experience in solving complex, interdisciplinary problems. They found that indigenous societies have been successful at solving societal problems thanks to the development of strong dialogue processes and a continued commitment to a holistic view of the world.

Insights

This approach taps into intuitive human abilities and interests as it prioritises lived stories and experiences. This means that it can be easily adopted and the results are often compelling. However, it can be hard to integrate the findings from these results into a holistic understanding of a system.

The most rigorous and powerful findings from this method require involvement of the particular actors you are considering. However, while it is hard to genuinely put yourself in someone else's situation, this can be used as an imaginative technique where people are asked to consider what a day/month/year in the life of an actor might be like. This can still illustrate important features of the system, albeit with much greater risks of inaccuracy.

Example Literature

Apgar, J. Marina, Alejandro Argumedo, and Will Allen. 2009. 'Building Transdisciplinarity for Managing Complexity: Lessons from Indigenous Practice'. *The International Journal of Interdisciplinary Social Sciences: Annual Review* 4(5):255–70. https://doi.org/10.18848/1833-1882/CGP/v04i05/52925.

Geertz, Clifford. 1972. 'Deep Play: Notes on the Balinese Cockfight'. *Daedalus* 101(1):1–37.

Break the system

Overview

One time honoured human approach to understanding things is to (accidentally or deliberately) break them. Figuring out how and why the thing broke is often a much more efficient way to understand key features of that thing that trying to build up an understanding from scratch. Toddlers, teenagers and tech start-ups (*Move fast and break things*) are all fans of this approach, which can be applied as a method for understanding complex systems.

As an analytic technique, this more often involves putting a system under (real or simulated) stress and see when and where it breaks. This often reveals important system dynamics or features and can be an easy way to explore the interaction of different scales. The breaking point in many systems is often something small rather than large.

Berche et al (2009) used this approach to study the resilience of public transport networks (PTNs) in 14 major cities by using simulations to subject each PTN to a set of 16 unique attack strategies. They found that particular network parameters, namely the Molloy-Reed parameters, were good indicators of stability. The results suggest more generally that those PTNs that are *scale-free* were more robust to random attacks but were more vulnerable to targeted attacks.

A version of this approach is embedded with the common method of *red teaming* within military analysis and wargaming. The object of setting up a *red team* is to play an adversarial role and try to put existing plans and capabilities under stress or break them.

Insights

This method is highly intuitive and simple to people to grasp. Usefully, people with expertise very often have clear ideas or intuitions about the potential failure points within a system that can be easily incorporated into analysis. As a counter-point to other forms of analysis it is often enjoyable or a useful change in ways of thinking.

However, it depends on having an existing understanding, or even model, of the system to work with. This might simply be an intuitive understanding but this means that it isn't normally suitable for an early stage of exploration or analysis.

One clear risk in using this method is that there are many ways a system can fail and not all of them provide useful information about the nature of the system. Overwhelming catastrophes, for example, are rarely useful cases of a system breaking to explore. The type of system failures, and the sources of it, need to be calibrated to the reasons for exploring a system. Generally, trying to understand failure points with minimal effort or input are likely to tell you more about the system.

Example Literature

Berche, B., C. von Ferber, T. Holovatch, and Yu. Holovatch. 2009. 'Resilience of Public Transport Networks against Attacks'. *The European Physical Journal B* 71(1):125–37. https://doi.org/10.1140/epjb/e2009-00291-3.

Sandoz, John. F. 2001. *Red Teaming: A Means to Military Transformation*.

Random selection

Overview

In contrast to approaches that seek to be comprehensive or representative, an alternate method is to pick out a small, tractable number of factors or entities that are widely dispersed across the whole complex system and map their relationships (possibly through intermediaries). This builds a somewhat idiosyncratic view of the system but is one that forces a deeper exploration of seemingly unconnected aspects of the system.

For example, to develop insights about a country's stability, you might pick the following and map out the influences and relationships: a small business in the country; a general in the country's military; a relevant international organisation; a particular commodity price; and public attitudes in a rival country. Forcing your analysis to think about how all of these might influence the others (and the various intermediaries) can help illuminate the system you are considering and other factors that you need to consider. Repeating this process for a few different sets will help build up a more holistic view of the system.

McCaffrey (2018) used an approach like this in his method for generating creative insights. This scaffolded cognition with a visual graph that allows for the discovery of obscure, but potentially important, features of the problem. The author suggests problem solvers can be aided in discovering these features by elaborating a bidirectional graph known as a Bi-net or BrainSwarm. Previous studies have shown that using the BrainSwarming method increases performance on insight puzzles which are typically taken to require creativity.

Insights

This method is not heavily covered in academic literature, partly as academic incentives encourage decisive, comprehensive theories. However, its demonstrated success in problem solving contexts suggests significant benefits in applied complex systems approaches.

Adopting this method requires being comfortable with only constructing partial understandings of the system that will be necessarily skewed by how it is set up. Being thoughtful about the set up, and repeating the process, can usefully build multiple different views on the system from differing perspectives. Forcing an analysis of how seemingly unconnected parts of the system might influence each other (perhaps indirectly) also forces more in depth thinking about the system without having to examine the whole system in depth.

Example Literature

McCaffrey, Tony. 2018. 'A Visual Representation to Quantitate, Diagnose, and Improve Creativity in Insight Problem Solving'. *The Journal of Creative Behavior* 52(1):52–65. https://doi.org/10.1002/jocb.132.

Empirical conceptual analysis

Overview

Complexity is often present in how different people understand a system, not just the system itself. Empirical conceptual analysis seeks to make explicit the features of how different people understand a system by getting them to articulate key terms and definitions. Gaining clarity on terms, via empirical testing not just thinking about it, often yields important insights about systems that are obscured when people use the same term but mean very different things by it.

A practical example of this approach can be found in Tomiyama et al. (2007), who explored the origins of the complexity involved in the production of mechatronics machines. The authors looked explicitly at the cases of two design failures: the rotary encoder design case and the AGV material handling system. They found that the design failures were largely due to a misunderstanding, or misclassification, of the interactions among design parameters at the physical phenomenon level. Key definitions needed to be refined via the intrinsic complexity of multi-disciplinarity to minimise design failures.

A more theoretic application is in Sytsma and Ozdemir's (2019) study on the "Hard Problem of Consciousness", which is the problem of explaining why our cognition and behaviour is accompanied by a qualitative or phenomenal aspect. Essentially, they deny that there is any problem in explaining why we believe that we are conscious because empirical explorations who that 'we' in the relevant sense, do not believe that. Importantly, they came to this conclusion by engaging with the domain of investigation, rather than directly studying the phenomena itself.

Insights

This is a relatively simple, tangible method to explore complex systems, if applied well. Notably, it does not explore the system directly but seeks to articulate how different actors perceive the system and their interests within it. Especially within social system, this can neatly reveal underlying system dynamics without large levels of analysis.

This approach is unlikely to reveal deep or hidden structures within a system as these are often features that no-one is explicitly aware of. It is better suited as an early stage method for starting to reveal key features of the system and can likely be used to help choose or design other approaches.

Example Literature

Sytsma, J., and E. Ozdemir. 2019. 'No Problem: Evidence That the Concept of Phenomenal Consciousness Is Not Widespread'. *Journal of Consciousness Studies* 26(9–10):241–56.

Tomiyama, T., V. D'Amelio, J. Urbanic, and W. ElMaraghy. 2007. 'Complexity of Multi-Disciplinary Design'. *CIRP Annals* 56(1):185–88. https://doi.org/10.1016/j.cirp.2007.05.044.

Comprehensive mapping

Overview

To many research mindsets, taking a comprehensive approach to understanding a system is intuitively the right place to start. It is therefore unsurprising that a range of researchers seek to take a comprehensive approach to a complex system, that is seek to identify all the relevant factors across all relevant scales and comprehensively map their relationships. However, due to the inherent complexity involved, this poses a range of difficult methodological and resourcing challenges. Projects that seek or claim to use a comprehensive mapping approach either focus on the methodology or tend to limit themselves in practice.

For example, Pahl-Wostl et. al. (2010) tackled the analysis of water systems via means of a systematic framework that works through a detailed seven stage process, further aggregated into three key phases. The paper provides the tools for a detailed understanding but in practice is limited by information constraints.

Similarly, Moon and Browne (2021) analysed the reef ecosystem via a mapping of four distinct phases. They were able to successfully track decisions, assumptions and existing data, whilst also engaging with the social components of the reef system. This, however, was a geographically limited system and the necessary inputs were extensive and detailed.

In a different example, Blackman et. al. (2017) aimed to use a comprehensive mapping approach to study the recovery process following the 2011 natural disasters in Japan and Christchurch. The study was successful in identifying three key factors that influence the transition from short-term to long-term recovery strategies. However, in practice, it had to limit itself to a sampling a range of key actors.

Insights

Where the resources, information and skills are available, taking a comprehensive mapping approach has significant strengths. The detail and nuance that can be developed through the process will provide a rich set of data and significant material for understanding the system and testing decisions. Importantly, it also forces a multi-disciplinary and multi-scale approach that considers a wide range of factors.

However, even in tightly constrained systems, the number of entities and factors to be mapped is generally very large and the resources and expertise required to do it well are large. Given the essentially multi-disciplinary nature of the project, good practices to get experts to collaborate across disciplinary boundaries are required.

At a broader level, a complete systems map is usually never achievable as gaps in knowledge or understanding will always exist. Comprehensive approaches like this are often liable to lead to a false sense of confidence in the outcomes of the analysis.

Example Literature

Blackman, Deborah, Hitomi Nakanishi, and Angela M. Benson. 2017. ‘Disaster Resilience as a Complex Problem: Why Linearity Is Not Applicable for Long-Term Recovery’. *Technological Forecasting and Social Change* 121:89–98. https://doi.org/10.1016/j.techfore.2016.09.018.

Moon, Katie, and Nicola K. Browne. 2021. ‘Developing Shared Qualitative Models for Complex Systems’. *Conservation Biology* 35(3):1039–50. https://doi.org/10.1111/cobi.13632.

Pahl-Wostl, Claudia, Georg Holtz, Britta Kastens, and Christian Knieper. 2010. ‘Analyzing Complex Water Governance Regimes: The Management and Transition Framework’. *Environmental Science & Policy* 13(7):571–81. https://doi.org/10.1016/j.envsci.2010.08.006.

Emergence

Overview

One of the fundamental insights of complex systems science is that systems can often have emergent properties that are not simple functions of the constituent parts of the system. Crowds, organisms, organisations, ecosystems and similar systems can have properties that are not directly reducible to the parts and can sometimes 'behave' as independent entities. It is therefore to be expected that research into complex systems focuses on emergence as a method. When looked at in terms of different scales, this involves examining smaller scales or lower levels for emergent properties or behaviours with which the scale of interest can be understood.

There are many papers that rely on emergence for their analysis, including classic papers in various disciplines, however the techniques involved varied with disciplines. For example, Menger (1892) is a classic paper in the history of economics. It provides a rationale for why money emerged as the universal currency. It was in the interests of local buyers and sellers to obtain goods with greater liquidity. As a result, market transactions converged toward money as it has the greatest liquidity. Note, this outcome arose from individuals simply pursuing their own diverse economic interests.

A similarly important paper in economics is Phillips (1958), which introduced produced a model describing the relationship between inflation and unemployment. Inflation and unemployment can be understood as emergent properties of regional and national economies that interact with each other. The Phillips Curve is one of the key models in macroeconomic theory.

In a different field, Taylor (1921) used an emergence approach to understand the energy dynamics and general structure of turbulent fluid flow. The Lagrangian approach pioneered by Taylor has been one of the key tools for analysing turbulent systems ever since.

A more explicit use of emergence is found in Hayek (1945), which explained how the divergent and erroneous information of economic actors can spontaneously generate an ordered outcome. The stable outcome emerges from the interactions between agents in the market that are informed by the price system.

Galam (2002) showed that public debates preceding a referendum are de facto anti-democratic. By using a simple one person-one argument model in which political discourse is shaped by the geometry of social life, e.g., within offices, bars, and houses, it is shown that an initial minority refusal can quickly evolve to widespread refusal. This is true even when the vast majority of the public would support change. This model provides an explanation for how the status quo can gain superiority from simple public debates.

Schelling's (1971) famous model of social segregation is an example of how suboptimal collective outcomes can emerge from complex systems. The model is comprised of individuals that are either white or black. He shows, that although both sets of individuals would prefer to live in a mixed neighbourhood, the stable state that emerges is characterised by clusters of homogenous individuals. It is the identification of tipping points and regions of stability in the system that is particularly interesting, as these dictate the final composition of individuals in the neighbourhood.

The aggregate motion of a flock of birds, a herd of land animals, or a school of fish in nature is seamlessly coordinated despite the absence of some centralised control agent. Reynolds (1987) produced a simple model of flocking birds to show how this is possible. The motion of each bird in the flock is modelled using three simple rules: collision avoidance, velocity matching, and flock centering. The flocking behaviour was seen to emerge from each individual bird simply following these three rules.

Lansing, Kremer, and Smuts (1998) employed a system-dependent model of natural selection to study the effects of environmental factors on the cropping patterns observed in the Balinese wet-rice fields. They find that the cropping patterns are directly influenced by two environmental factors: the control of pests and the management of irrigation flows. This conclusion is supported by the use of an agent-based model built on these assumptions that is able to successfully reproduce the structure of cropping patterns observed in real life.

Insights

Taking an explicit 'emergence' approach focuses the understanding squarely on one of the key complex system dynamics which most clearly diverges from traditional reductionist approaches. Where it is successful, if very often delivers new insights and techniques that make the complex system more understandable and explainable. It can identify new dynamics, factors or forces at play that need to be understood.

One challenge with this approach – understood as a method - is that, to date in the literature, there is no repeatable process or method to use. It tends to require close understanding of the system at play and human ingenuity in identifying insights. This may change with further research.

There is also a built-in assumption of bottom-up causation in this method. While this is works with some systems, in many others there are feedback loops between different scales. For example, international bodies and systems influence nation state behaviour, which in turn then changes the international level. Emergence ignores the possibility of factors from higher levels influencing or causing change at lower levels.

Example Literature

Galam, S. 2002. 'Minority Opinion Spreading in Random Geometry'. *The European Physical Journal B - Condensed Matter and Complex Systems* 25(4):403–6. https://doi.org/10.1140/epjb/e20020045.

Hayek, F. A. 1945. 'The Use of Knowledge in Society'. *The American Economic Review* 35(4):519–30.

Lansing, JS, JN Kremer, and BB Smuts. 1998. 'System-Dependent Selection, Ecological Feedback and the Emergence of Functional Structure in Ecosystems'. *Journal of Theoretical Biology* 192(3):377–91. https://doi.org/10.1006/jtbi.1998.0664.

Menger, Karl. 1892. 'On the Origin of Money'. *The Economic Journal* 2(6):239–55. https://doi.org/10.2307/2956146.

Phillips, A. W. 1958. 'The Relation Between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861–19571'. *Economica* 25(100):283–99. https://doi.org/10.1111/j.1468-0335.1958.tb00003.x.

Reynolds, Craig W. 1987. 'Flocks, Herds and Schools: A Distributed Behavioral Model'. *ACM SIGGRAPH Computer Graphics* 21(4):25–34. https://doi.org/10.1145/37402.37406.

Schelling, Thomas C. 1971. 'Dynamic Models of Segregation'. *The Journal of Mathematical Sociology* 1(2):143–86. https://doi.org/10.1080/0022250X.1971.9989794.

Taylor, G. I. 1922. 'Diffusion by Continuous Movements'. *Proceedings of the London Mathematical Society* s2-20(1):196–212. https://doi.org/10.1112/plms/s2-20.1.196.

Complexity watch

Overview

Most of the methods used to understand complexity start with the elements, relationships, factors or properties within the system and try to understand their interactions to explore the system and identify complexities. An alternative approach draws explicitly on complex systems science and focuses on known properties of complex systems, such as emergence, nonlinearity, self-organisation.

This approach examines systems to look for typical complex systems behaviours. This then illuminates the structure of the system and helps understand how it might change or evolve. It also narrows the focus in on complex dynamics that can be further investigated.

For example, Filotas et. al. (2014) developed a conceptual framework for managing forest ecosystem using eight common properties of complex systems. The introduction of cross-scale interactions makes this framework more flexible and robust than the traditional isolated subsystem approaches used in forest ecology.

van Rooj et. al. (2013) explored the dynamics of human decision making under uncertainty. They argue that the phase transitions and multistability that characterises hysteresis necessitates the use of a nonlinear model and they explored the possibility of using a simple nonlinear dynamical system to capture the relevant dynamics. On a similar topic, Maylor et. al. (2001) tested the *scale-invariance* of memory retrieval. They found that memory retrieval was highly similar across different time periods of a day, week or year. The existence of *scale-invariance* is an indicator of complexity in human memory retrieval.

Ilachinski (1996) developed an eight-tiered conceptual framework to study land warfare as a complex system. The framework implements a wide range of strategies observed elsewhere in the literature. For instance, the development of network models and genetic algorithms to study combat, the identification of patterns in real-world combat, and the extension of existing warfare models to include nonlinear dynamics are all found in the framework.

Stephen et al. (2009) made use of entropy measurements and the presence of power law behaviour to study the emergence of new cognitive structures. The researchers used time series of changes in point-of-gaze to test whether the discovery of a particular mathematical relation by participants could be understood as a phase transition. They found that the key parameters of entropy and power-law behaviour were predictive of mathematical discovery. The result of this study is particularly interesting as the conventional fixation measures of reaction time, accuracy, number of fixations and the duration of fixations all failed to predict the onset of mathematical insight!

Duit et al. (2010) approached the multilevel governance challenges posed by social-ecological systems from a "resilience" perspective. This method involves understanding how these systems can withstand, and develop from, disturbances and change. The authors study a range of systems from the resilience perspective, including the robustness of international environmental regimes and the 'framing' of health risks at the national and international level.

Baynes (2009) provided an overview of how complex approaches have been applied to urban development. The author in their application of the method did a historical relay of different methods, highlighting their strengths and weaknesses, and where and when they are best suited. They concluded that housing problems often exhibit complex behaviour, and the current research agenda could be enriched by making use of the dynamic models and tools of complexity science.

Insights

This approach has been very popular within the academic literature. Starting with what we know about complexity can be a powerful way of quickly identifying features of a system and doesn't require bottom up mapping or in-depth investigations. It forces researchers and analysts to explicitly consider complexity and disrupts normal patterns of thinking. These approaches can allow analysis to quickly focus on key patterns, structures or emergent behaviours.

It does, however, require analysts and researchers to be sufficiently familiar with complexity to spot common patterns and distinguish them from other structures. This reduces its usefulness outside specialist or those with considerable expertise. Also, in relying heavily on analyst skill and expertise, it is likely more open to common biases and will require more external structure or a more diverse range of people involved to minimise this. The risk is that it becomes normative rather than descriptive of the system as analysts are likely to more easily spot the patterns or behaviours they expect or want to see.

Example Literature

Baynes, Timothy M. 2009. 'Complexity in Urban Development and Management'. *Journal of Industrial Ecology* 13(2):214–27. https://doi.org/10.1111/j.1530-9290.2009.00123.x.

Duit, Andreas, Victor Galaz, Katarina Eckerberg, and Jonas Ebbesson. 2010. 'Governance, Complexity, and Resilience'. *Global Environmental Change* 20(3):363–68. https://doi.org/10.1016/j.gloenvcha.2010.04.006.

Filotas, Elise, Lael Parrott, Philip J. Burton, Robin L. Chazdon, K. David Coates, Lluís Coll, Sybille Haeussler, Kathy Martin, Susanna Nocentini, Klaus J. Puettmann, Francis E. Putz, Suzanne W. Simard, and Christian Messier. 2014. 'Viewing Forests through the Lens of Complex Systems Science'. *Ecosphere* 5(1):art1. https://doi.org/10.1890/ES13-00182.1.

Ilachinski, Andrew. 1996. *Land Warfare and Complexity, Part II: An Assessment of the Applicability of Nonlinear Dynamics and Complex Systems Theory to the Study of Land Warfare*. CENTER FOR NAVAL ANALYSES ALEXANDRIA VA.

Malamud, Bruce D., Gleb Morein, and Donald L. Turcotte. 1998. 'Forest Fires: An Example of Self-Organized Critical Behavior'. *Science* 281(5384):1840–42. https://doi.org/10.1126/science.281.5384.1840.

Maylor, Elizabeth A., Nick Chater, and Gordon D. A. Brown. 2001. 'Scale Invariance in the Retrieval of Retrospective and Prospective Memories'. *Psychonomic Bulletin & Review* 8(1):162–67. https://doi.org/10.3758/BF03196153.

Stephen, Damian G., Rebecca A. Boncoddo, James S. Magnuson, and James A. Dixon. 2009. 'The Dynamics of Insight: Mathematical Discovery as a Phase Transition'. *Memory & Cognition* 37(8):1132–49. https://doi.org/10.3758/MC.37.8.1132.

van Rooij, Marieke M. J. W., Luis H. Favela, MaryLauren Malone, and Michael J. Richardson. 2013. 'Modeling the Dynamics of Risky Choice'. *Ecological Psychology* 25(3):293–303. https://doi.org/10.1080/10407413.2013.810502

Walker, Guy H., Neville A. Stanton, Paul M. Salmon, Daniel P. Jenkins, and Laura Rafferty. 2010. 'Translating Concepts of Complexity to the Field of Ergonomics'. *Ergonomics* 53(10):1175–86. https://doi.org/10.1080/00140139.2010.513453.

Element Property Dynamics

Overview

In some systems, all the relevant elements or entities have common properties that are easier to track than the elements themselves. By identifying, tracking, or analysing these properties, the broader complex system can sometimes be understood in ways that aren't possible by means of looking at the elements. This differs from approaches that focus on relationships between elements or emergent properties of systems.

In the literature review to date, this has been mostly found when looking at physical systems as there are many established methods for tracking physical properties, like energy, entropy, or various phase states. For example, Chaisson (2015) proposed a model for understanding the evolution of the universe in terms of its complexity. Using energy rate density as a proxy for complexity, he produced a general trend that suggests that the complexity in the universe has increased across time.

More famously given the 2021 Nobel Prize, Parisi (1979) generated a model that sufficiently describes the phase of "spin glasses". This result was produced from local properties by focussing on the probability distributions of the many meta-stable states of the spin glass and has applications to other systems with many *near-optimal* solutions. More broadly, it is an interesting example of systems with many locally stable solutions but not global stability.

Kuran (1989) developed a model to explain unexpected revolutions without relying on hindsight. He assumes that individuals have two political preferences: one public and one private. Working from this assumption, a *revolutionary potential* term is developed. By tracking this, one could hypothetically gauge how likely a society is to revolt following a minor shock. However, due to what is involved with calculating this term, the model has little predictive capability.

However, this approach is likely to be applicable to a much broader set of complex systems. For example, a workplace could be analysed by tracking workload, diary availability or individual volumes of emails.

Insights

Where properties that fit the requirements for this method exist – that is they hold for elements in the system and interact through the system – and they are measurable or quantifiable (at least in theory), then this method provides a promising way of understanding the complexity of the system. It allows the complex behaviour to emerge from the analysis and be investigated with a range of traditional research methods.

The strengths of this approach, however, only apply if the right sort of properties exist. It seems possible that properties like this exist in systems and disciplines where it hasn't traditionally been applied. But it also is not likely to be universally applicable. Narrowing the focus in on limited trackable properties will also likely mask complexities and there are risks that focusing on properties of elements will prevent insights about emergence.

Example Literature

Chaisson, Eric J. 2015. 'Energy Flows in Low-Entropy Complex Systems'. *Entropy* 17(12):8007–18. https://doi.org/10.3390/e17127857.

Kuran, Timur. 1989. 'Sparks and Prairie Fires: A Theory of Unanticipated Political Revolution'. *Public Choice* 61(1):41–74. https://doi.org/10.1007/BF00116762.

Parisi, G. 1979. 'Infinite Number of Order Parameters for Spin-Glasses'. *Physical Review Letters* 43(23):1754–56. https://doi.org/10.1103/PhysRevLett.43.1754.

Reference Appendix B: Overview of building techniques

The range of techniques covered for building an understanding of, or modelling, a complex system are more consistently conceptualised and applied within a range of academic and other literature. This reference section therefore focuses on a simple explanation of the technique and links to references for further reading. It does not cover the techniques in the same detail as for the 'Explore' techniques.

Discursive approaches

Games or Simulations

Codify key insights about a system as rules of a game and get people with relevant expertise to play out the game as a way of modelling the system, communicating features and building greater understanding. Especially within military studies, there is an extensive literature on these techniques, although some approaches can remove rather than capture complexity.

Example Literature:

Brown, G. 2019. *Successful Professional Wargames: A Practitioner's Handbook [by Graham Longley Brown]*. edited by J. Curry. London: History of Wargaming Project.

Perla, Peter. 1990. *Art of Wargaming*. Annapolis, Md: Naval Institute Press.

General Principles

A simple way to capture and communicate the behaviour of a complex system can be to articulate a number of general (but likely not universal) principles that explain typical system behaviour. Any paper that reports "X key findings" or "Y features of some situation" are intuitively using this approach. It also has deep roots in lived human experience as all cultures rely on a range of proverbs or sayings that count as general principles to help people navigate complexity.

Metaphors

Very disparate complex systems often have similar properties or behave in similar ways due to similar system structures. This means that metaphors that explain unfamiliar systems in terms of familiar ones can powerfully capture and communicate insights and system properties. This has been adopted in a range of research and literature and often the most iconic publications draw heavily on metaphor. A good example is Carl von Clausewitz's *On War*.

Example Literature:

Clausewitz, Carl von, Michael Howard, and Peter Paret. 1976. On War. Princeton, N.J: Princeton University Press.

Scenarios

Case studies, personas and scenarios are common ways of explaining more complex systems. It is generally easier to capture and articulate complexity within a narrative format than a standard analytic one. Narratives depend on tension, choices and conflict – which shape the account towards more complexity. Scenarios can be focused on different potential system states, or alternatively describe different ways that the system could start evolving or be put under stress.

Formal approaches

A good overview of typical formal approaches to modelling complex systems can be found in:

Boccara, Nino. 2010. *Modeling Complex Systems*. New York, NY: Springer New York.

Agent-based models

An agent is a thing that has some state which is modified through mutual interactions with other agents. An agent-based model (ABM) has the following key components: a set of agents and their states, the environment they exist in, and the laws that govern the interactions of agents. The state of an agent can be simple, e.g., a position or a binary response. The result of these components is a system that contains autonomous decision-making agents and can generate unique behaviour.

Example Literature:

Malamud, Bruce D., Gleb Morein, and Donald L. Turcotte. 1998. 'Forest Fires: An Example of Self-Organized Critical Behavior'. *Science* 281(5384):1840–42. https://doi.org/10.1126/science.281.5384.1840.

Mueller, Michel G., and Peter de Haan. 2009. 'How Much Do Incentives Affect Car Purchase? Agent-Based Microsimulation of Consumer Choice of New Cars — Part I: Model Structure, Simulation of Bounded Rationality, and Model Validation'. *Energy Policy* 37(3):1072–82. https://doi.org/10.1016/j.enpol.2008.11.002.

Nagel, Kai, and Michael Schreckenberg. 1992. 'A Cellular Automaton Model for Freeway Traffic'. *Journal de Physique I* 2(12):2221–29. https://doi.org/10.1051/jp1:1992277.

Reynolds, Craig W. 1987. 'Flocks, Herds and Schools: A Distributed Behavioral Model'. *ACM SIGGRAPH Computer Graphics* 21(4):25–34. https://doi.org/10.1145/37402.37406.

Network Models

Build a formal model based on properties of the relationships between entities – constructed as a network - rather than the entities themselves. Focusing on a few key network parameters (e.g., characteristic path length, clustering coefficient, and connectivity) can capture important structural features of the system and allow the use of a range of mathematical and computing techniques.

Example Literature:

Boss, Michael, Helmut Elsinger, Martin Summer, and Stefan Thurner. 2004. 'An Empirical Analysis of the Network Structure of the Austrian Interbank Market'. *Financial Stability Report* (7):77–87. https://doi.org/10.1080/14697680400020325.

Ferrer i Cancho, Ramon, Oliver Riordan, and Béla Bollobás. 2005. 'The Consequences of Zipf's Law for Syntax and Symbolic Reference'. *Proceedings of the Royal Society B: Biological Sciences* 272(1562):561–65. https://doi.org/10.1098/rspb.2004.2957.

Quintana, Ignacio Ojea, Marc Cheong, Mark Alfano, Ritsaart Reimann, and Colin Klein. 2022. 'Automated Clustering of COVID-19 Anti-Vaccine Discourse on Twitter'. https://doi.org/10.48550/ARXIV.2203.01549.

Storrs, Katherine R., and Guido Maiello. 2020. 'A Model for Neural Network Modeling in Neuroscience'. *Journal of Neuroscience* 40(37):7010–12. https://doi.org/10.1523/JNEUROSCI.1205-20.2020.

Power Law Distributions

Empirically, complex system behaviour often reflects various mathematical power laws. The power law states that a relative change in one quantity results in a proportional relative change in another. Power laws are typically represented with the following equation: $Y = AX^B$, where Y is the quantity to be measured, X is the quantity to be varied, and A and B are constants. Power laws can be a simple way to model systems, especially when an unfamiliar system is structurally similar to a more familiar system.

Example Literature:

Pareto, Vilfredo. 1964. *Cours d'économie politique*. Librairie Droz.

Willis, J. C., and G. Udny Yule. 1922. 'Some Statistics of Evolution and Geographical Distribution in Plants and Animals, and Their Significance'. *Nature* 109(2728):177–79. https://doi.org/10.1038/109177a0.

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Differential and Recurrence Equations

Differential equations and recurrence equations can be used to create dynamic models of a system. They can model how the unique state of a system evolves temporally according to predefined rules. Differential equations deal with continuous, smooth processes. Recurrence equations deal with discrete, stepwise processes.

Example Literature:

Kermack, William Ogilvy, A. G. McKendrick, and Gilbert Thomas Walker. 1927. 'A Contribution to the Mathematical Theory of Epidemics'. *Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character* 115(772):700–721. https://doi.org/10.1098/rspa.1927.0118.

Lotka, Alfred J. 1910. 'Contribution to the Theory of Periodic Reactions'. *The Journal of Physical Chemistry* 14(3):271–74. https://doi.org/10.1021/j150111a004.

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Subsystem modelling

Build a model for the whole system by focusing on the interactions between subsystems with their own (emergent) properties, rather than one connected system. This might build on models of the subsystems, or just their emergent properties.

Example Literature:

Garira, Winston. 2020. 'The Research and Development Process for Multiscale Models of Infectious Disease Systems'. *PLOS Computational Biology* 16(4):e1007734. https://doi.org/10.1371/journal.pcbi.1007734.

Raabe, Dierk, Matthias Scheffler, Kurt Kremer, Walter Thiel, Jörg Neugebauer, and Martin Jansen. 2009. *Multi-Scale Modeling in Materials Science and Engineering*.

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